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Axial Turbines

Blade and Disk Stresses, and Vibrations

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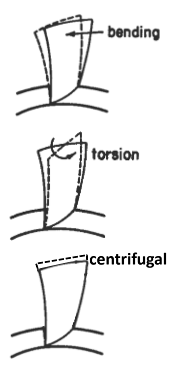
Turbine Blade Stresses

- Stresses on turbine blades
  - thermal stresses (temperature gradients)
  - bending stresses (gas loads)
  - torsion (gas loads)
  - centrifugal stresses (spinning rotor)
- Cycling
  - thermal
  - mechanical, including vibrations
- Dominant contribution in rotor and rotor disk
  - **centrifugal stresses**
    - limits N, h, r_m,…
- AND lower stresses ⇒ longer lifetime

also applies to compressors



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Creep Rupture Strength

- For rotor blades, the limiting specific tensile strength $(\sigma/\rho)_{max}$ (known as *allowable strength-to-weight ratio*) is based on **creep rupture strength**
  - maximum tensile σ mat'l can tolerate w/o failure due to creep for σ const. for given time at given T
- Typically use 50% of this as limit
- For compressor rotor, low T ⇒ different matls can be used

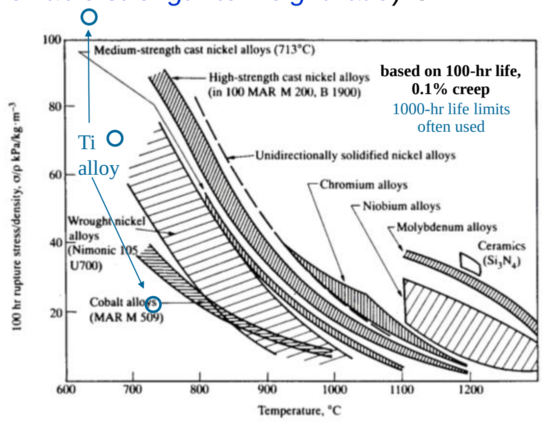

FIGURE 8.14 Variations in specific rupture strength (100 hr) with service temperature for various classes of heat-resistant materials. (Courtesy Imarigeon [10].)

Mechanics and Thermodynamics of Propulsion, Hill and Peterson

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Creep Time-Temperature Tradeoff

- Increasing temperature results in decreased time a given material can tolerate a fixed σ before hitting creep limit
- Tradeoff in temperature and time captured in the **Larson-Miller parameter**

$$T_{abs}(C + \log t_{hrs})$$

  - C~25 for turbine disk alloys (Wilson and Korakianitis, 1998)
  - example: for disk material that hits creep limit in 1000 hr at 1400 K
    - how long if raised to 1500 K? ***only 13.6 hrs!!***

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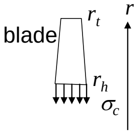
Centrifugal Stresses



- As noted before, largest rotor stress contribution is centrifugal
- For differential blade element at radius r, the differential centrifugal force is

$$dF_c = -\Omega^2 r\,dm = -\Omega^2 r \rho_{blade}(r) A_{blade}(r)\,dr$$

- So centrifugal force at r $\quad F_c(r) = -\Omega^2 \int_{r_t}^{r} \rho_{blade}(r) A_{blade}(r)\, r\,dr$
- Centrifugal stress is $\quad \sigma_c(r) = F_c(r)/A(r)$
- Maximum σ_c occurs at blade hub (root)

$$\sigma_{c,\max} = \frac{F_{c,h}}{A_h} = \Omega^2 \int_{r_h}^{r_t} \rho_{blade}(r) \frac{A_{blade}(r)}{A_h}\, r\,dr$$

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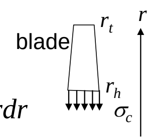
Blade Tapering



- For solid blade of uniform composition (or neglecting cooling passage volume)

$$\frac{\sigma_{c,\max}}{\rho_{blade}} = \Omega^2 \underbrace{\int_{r_h}^{r_t} \frac{A_{blade}(r)}{A_h}\, r\,dr}_{\equiv\ taper}$$

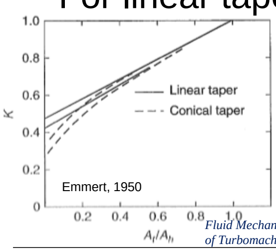
- For untapered blades

$$\frac{\sigma_{c,\max}}{\rho_{blade}} = \Omega^2 \left(r_t^2 - r_h^2\right) = U_t^2 \left[1 - \left(\frac{r_h}{r_t}\right)^2\right]$$

one reason to limit tip speeds

- For linear taper

$$\frac{\sigma_{c,\max}}{\rho_{blade}} = \Omega^2 \int_{r_h}^{r_t} \left[1 - \frac{r - r_h}{r_t - r_h}\left(1 - \frac{A_t}{A_h}\right)\right] r\,dr$$



Fluid Mechanics and Thermodynamics of Turbomachinery, Dixon and Hall

$$K \equiv \frac{\sigma_{c,\max,tapered}}{\sigma_{c,\max,untapered}}$$

tapering blades reduces centrifugal stress

$A_t/A_h \equiv$ taper ratio
turbine rotors **~0.2-1** (1=untapered)
compressor rotors **~0.5-1**

Centrifugal Stresses

- Integrating linear taper case

$$\frac{\sigma_{c,\max}}{\rho_{blade}\Omega^2}=\frac{r_t^2-r_h^2}{2}-\left(1-\frac{A_t}{A_h}\right)\frac{1}{r_t-r_h}\left[\left(\frac{r_t^3}{3}-\frac{r_t^2 r_h}{2}\right)-\left(\frac{r_h^3}{3}-\frac{r_h^2 r_h}{2}\right)\right]$$

$$=\frac{r_t^2-r_h^2}{2}-\left(1-\frac{A_t}{A_h}\right)\frac{1}{r_t-r_h}\left[\frac{2r_t^3+r_h^3-3r_hr_t^2}{6}\right]$$

$$(r_t-r_h)\frac{r_t+r_h}{2}$$

$$\left[\frac{(r_t-r_h)^2(2r_t+r_h)}{6}\right]$$

$$=(r_t-r_h)\left[\left(\frac{r_t+r_h}{2}\right)+\left(1+\frac{A_t}{A_h}\right)\left(\frac{2r_t+r_h}{6}\right)\right]$$

$$=(r_t-r_h)\left[\left(\frac{3(r_t+r_h)}{6}-\frac{2r_t+r_h}{6}\right)+\frac{A_t}{A_h}\left(\frac{2r_t+r_h}{6}\right)\right]$$

$$=(r_t-r_h)\left[\left(\frac{r_t+2r_h}{6}\right)+\frac{A_t}{A_h}\left(\frac{2r_t+r_h}{6}\right)\right]$$

Centrifugal Stresses

- So

(II.59a) $$\frac{\sigma_{c,\max}}{\rho_{blade}}=\Omega^2(r_t-r_h)\left[\left(\frac{r_t+2r_h}{6}\right)+\frac{A_t}{A_h}\left(\frac{2r_t+r_h}{6}\right)\right]$$

  – assuming $r_t+2r_h\cong r_h+2r_t\cong 3r_m$

  and (axial) ***Flow Area*** $A_z\cong 2\pi r_m(r_t-r_h)$

(II.59b) $$\frac{\sigma_{c,\max}}{\rho_{blade}}\cong\frac{A_z\Omega^2}{4\pi}\left(1+\frac{A_t}{A_h}\right)$$

for given material, centrifugal stress scales with blade angular speed, flow area and taper ratio

- Leads to what is known as the **AN^2 rule**
  – design limit for maximum allowed A_zN^2
  – traditional turbines/mat'ls. typical

for a turbine material at max temp.

$$A_zN^2|_{max}=0.5\text{–}10\times10^{10}\ in^2\ RPM^2$$
$$=0.3-6\times10^7\ m^2\ RPM^2$$

Maximum Hub Speed Limits

- Maximum allowed rotational speed leads to maximum blade speeds
- For example at turbine rotor hub

$$U_{h,\max} = r_h N_{\max}\, \pi/30$$

  - typical (conventional) values $U_{h,max}$
    - HPT: 300 – 500 m/s (1000 – 1500 ft/s)
    - LPT: 150 – 300 m/s (500 – 1000 ft/s)

Bending Stresses

- Simple model for cantilevered blade in a stage under aerodynamic loading*

$$\frac{\sigma_{bend}}{p_{avg}} \approx \frac{c_z}{U_{tip}} \frac{|\Delta h_{o1,3}|}{c_p T_{o1}} \frac{1}{2\sigma} \left(\frac{r_{tip}}{t_{\max}}\right)^2 \quad \textbf{(II.60)}$$

- Bending stresses increase with
  - higher flow coefficient (at tip)
  - more specific stage work (loading)
  - lower solidity
  - increasing tip radius-to-thickness ratio

after Kerrebrock, Aircraft Engines and Gas Turbines, 2nd ed., MIT Press (1992); and Farokhi, Aircraft Propulsion

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Example: Turbine Rotor Stresses

- **Given**:
  - turbine stage design from previous example(s)
  - additionally h =5.0cm, taper ratio of 1/2, t_{max}/h =1/6, p_{rotor}=7.5bar
- **Find:**
  1. Required creep rupture strength-to-weight ratio
  2. Rotor bending stress
- **Assume:** same as previous (c_z const, tpg/cpg,…)

r_m= 0.45 m U_m=526 m/s

MW=28.7 N=11,160 rpm

γ=1.33

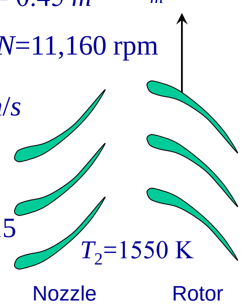
c_1=c_z=447 m/s

T_{o1}= 1800 K

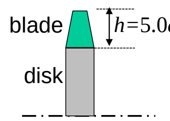
ϕ_m = 0.85

ψ = –1.15

T_2=1550 K



σ_r = 1.6



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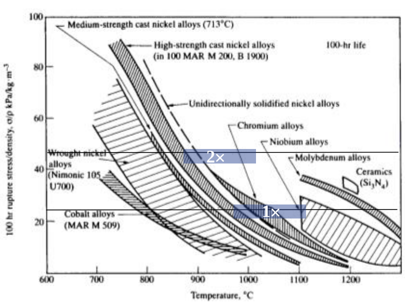
Example: Turbine Rotor Stresses

- **Required rupture strength-to-weight ratio**
  - needed to match centrifugal stress
    - from II.59b $\dfrac{\sigma_c}{\rho_{blade}} \cong \Omega^2 \dfrac{A_z}{4\pi}\left(1+\dfrac{A_t}{A_h}\right) = \left(N\dfrac{\pi}{30}\right)^2 \dfrac{r_m h}{2}\left(1+\dfrac{A_t}{A_h}\right)$

$$= \left(11160\frac{\pi}{30}s^{-1}\right)^2 \frac{0.45(0.050)m^2}{2}(1+0.5)$$

A_z = 0.14 m^2

$$= 2.31\times10^4 \frac{m^2}{s^2}\frac{kg/m}{kg/m} = \boxed{23.1\frac{kPa}{kg/m^3}}$$



$\Rightarrow$ max T_{rotor} < 1200-1350 K depending on mat'l. used

$\Rightarrow$ T_2=1550 K, so will need cooling/TBC

also $A_z N^2 \sim 2\times10^7$; within typical range

Example: Turbine Rotor Stresses

- **Rotor bending stress**
  - from II.60 $\dfrac{\sigma_{bend}}{p_{avg}} \approx \dfrac{c_z}{U_{tip}} \dfrac{\left|\Delta h_{o1,3}\right|}{c_p T_{o1}} \dfrac{1}{2\sigma} \left(\dfrac{r_{tip}}{t_{\max}}\right)^2$

$$\frac{c_z}{U_{tip}} = \phi_m \frac{U_m}{U_{tip}} = \phi_m \frac{r_m}{r_{tip}} \approx \phi_m \left(1+\frac{h/2}{r_m}\right)^{-1} = 0.85\left(1+\frac{0.025}{0.45}\right)^{-1} = 0.805$$

$$\frac{\left|\Delta h_{o1,3}\right|}{c_p T_{o1}} = \frac{\left|\psi U_m^2\right|}{\gamma/(\gamma-1) R T_{o1}} = \frac{\left|-1.15(526/s)^2\right|}{4(289.7\,J/kgK)1800K} = 0.151$$

$$\frac{r_{tip}}{t_{\max}} \approx \frac{r_m + h/2}{t_{\max}} = \frac{r_m/h + 1/2}{t_{\max}/h} = \frac{0.45/0.050 + 1/2}{1/6} = 57$$

$$\sigma_{bend} \approx (7.5bar)0.805(0.151)\frac{1}{3.2}(57)^2 = \boxed{92MPa}$$

comparing to centrifugal stress, using $\rho_{\text{steel alloys}}$ ~ 8200 kg/m^3
σ_c ~ 23.1 (8200) *kPa* ~ 190 *MPa* $\Rightarrow$ **σ_c ~ 2 σ_{bend}** *at our pressure*

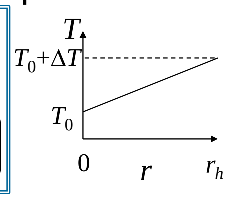
Thermal Stresses

- Based on thermal strains in material caused by a temperature difference ΔT

$$\sigma_t = E\varepsilon_t = E\alpha\Delta T$$

E = modulus of elasticity
ε_t = thermal strain
α = coeff. of linear thermal expansion

- For turbine disk, simple model is constant thickness, no central hole, linear temperature profile in r
  - radial stress $\sigma_{t,r} = \dfrac{\alpha E \Delta T}{3}\left(1-\dfrac{r}{r_h}\right)$
  - tangential stress $\sigma_{t,\theta} = \dfrac{\alpha E \Delta T}{3}\left(1-2\dfrac{r}{r_h}\right)$



maximum radial thermal stress at r_{min} (disk center)

(II.61)

Example: Disk Thermal Stresses

- Nickel-alloy disk $\sigma_{t,r} = \frac{\alpha E \Delta T}{3}\left(1 - \frac{r}{r_h}\right)$ $\sigma_{t,\theta} = \frac{\alpha E \Delta T}{3}\left(1 - 2\frac{r}{r_h}\right)$
  - $\alpha \sim 10.2\times10^{-6}$ in/in °F at 1400°F (1033K)
  - $E \sim 20.5\times10^{6}$ psi at 1400°F
  - ΔT
    - 200 °F (100 K)
  - @ $r = 0$, $\sigma_{t,r} = \sigma_{t,\theta} \cong 14\ ksi = 2\ Mpa$

    acceptable, only 3% of yield strength (0.2% offset) at 900 K
  - @ $r = r_h$, $\sigma_{t,r} = 0$, $\sigma_{t,\theta} \cong -2\ MPa$

to reduce thermal stresses, need lower ΔT
⇒ disk material should have high thermal conductivity, e.g., nickel alloys

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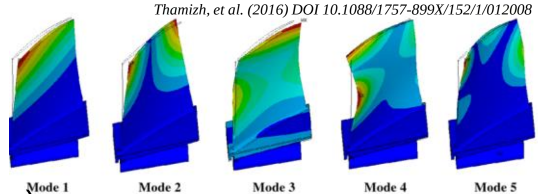
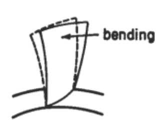
Blade Vibrations

- Blades (nozzle or rotor) essentially cantilevered beams
- Exhibit resonant (vibrational) mode shapes, each with its own natural frequency
  - bending (1^{st}, 2^{nd},..)
  - torsional (1^{st},…)
  - coupled bending/torsion
- Disks and shafts also have vibrational resonances
- Need to avoid running turbine at rotational speeds that "match" these resonances

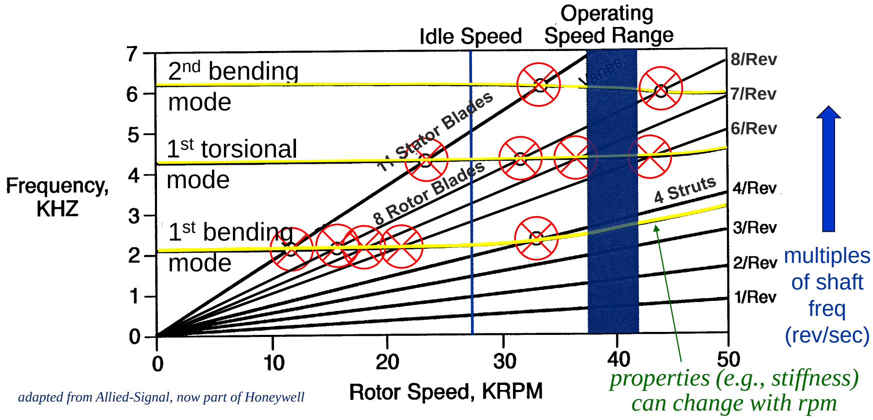
Thamizh, et al. (2016) DOI 10.1088/1757-899X/152/1/012008



increasing frequency



Campbell Diagram (Comp. Rotor)



adapted from Allied-Signal, now part of Honeywell

- Avoid design/operation at shaft speed that has integer multiples (up to # stators, # rotor, # struts) that match any structural resonances
- Same considerations true for turbine

Comments on Blade Cooling

- Cooling of at least 1st stage HPT nozzle and rotor typically required to maintain sufficient material strength
  – high temp., turbines, sometimes 2nd stage too
- Maximum stresses typically at rotor hub
- Most difficult regions to cool are blade tip and trailing edge
- Maximum heat load near blade leading edge (stagnation region)
  – rotor sees lower stagnation temperature due to relative motion