Turbomachinery

Axial Compressors

Cascade Flow Angles and Velocity Triangles
Compressor Analysis

- From Euler turbomachinery (conservation) equations need to understand change in tangential velocity to relate to forces on blades and power

\[
T = \dot{m} \Delta (r \omega) \\
\dot{W}/\dot{m} = \Delta (u_\omega)
\]

- Analyze cascade flow to find values for \(c_\theta\) (azimuthal or swirl velocity) through a compressor stage

Compressor Cascade Analysis

- Compressor stage consists of moving (rotor) and stationary (stator) blades
  - state change across each blade row
    1\(\rightarrow\)2 rotor
    2\(\rightarrow\)3 stator
- Use flow angles and velocity triangles to visualize transition between reference frames
Compressor (Cascade) Flow Angles

- Recall, two reference frames for fluid velocity
  - engine’s \( \vec{c} \)
  - blade’s \( \vec{w} \)
- Difference due to rotor motion
  \[ \vec{w} = \vec{c} - \vec{u} \]
  \[ \vec{c} = \vec{w} + \vec{u} \]
- For cascade flow (no radial vel. component)
  - use \( u \) to define +\( \theta \) dir.
  - define angles for each ref. frame (\( \alpha \) and \( \beta \))

To examine airfoil behavior, best to define flow angle in blade’s reference frame

For rotor, use velocities/angles in rotating frame
- \( w, \beta \)
- net flow turning angle,
  \[ \beta_1 - \beta_2 \] (II.9)
- will equal \( \phi_r \)
  if flow matches blade angles
  generally \( \neq \), but could be close
Compressor (Cascade) Flow Angles

- For **stator**, use engine reference frame
  - \( c, \alpha \)
  - net flow
    - turning angle, \[ \alpha_2 - \alpha_3 \] (II.10)
    - will equal \( \varphi_s \) if flow matches blade angles
      - generally \( \neq \), but could be close

Flow vs. Blade Angles

- How well flow matches blade ("metal") angles is given by
  - inflow: **incidence** \((i)\)
    \[ i \equiv -(\beta_1 - \chi_1) \] (rotor)
    \[ i \equiv \alpha_2 - \chi_{2s} \] (stator)
  - outflow: **deviation** \((\delta)\)
    \[ \delta \equiv -(\beta_2 - \chi_{2r}) \] (rotor)
    \[ \delta \equiv \alpha_3 - \chi_3 \] (stator)
Velocity Triangles

- Rotor motion in $\theta$ direction, so ref. frame change has no effect on other directions $\Rightarrow \begin{align*}
  \vec{w}_i &= \vec{c}_z i 
\end{align*}$ (II.13)

- Rotor vel. constant in cascade flow (like fixed $r$), and let $|\vec{u}_i| = U$
  $\Rightarrow \begin{align*}
  \vec{c}_i &= \vec{w}_i + \vec{u}
\end{align*}$ as shown here, $w_{\theta} < 0$

- Also have general trigonometric relations

  - e.g.,
    $\begin{align*}
      c_{\theta_1} &= c_i \sin \alpha_i = c_{z_1} \tan \alpha_i \\
      w_{\theta_1} &= w_i \sin \beta_i = w_{z_1} \tan \beta_i \\
      w_{z_1} &= w_i \cos \beta_i = c_{z_1} = c_i \cos \alpha_i
    \end{align*}$ (II.15)

Velocity Triangle Analysis

- Apply (II.12-14) to both blade rows of compressor stage
- 1: rotor inlet
  $\begin{align*}
    (13) \Rightarrow c_{\theta_1} &= c_{z_1} \tan \alpha_1 \\
    (14) \Rightarrow c_{\theta_2} &= U + w_{\theta_2}
  \end{align*}$ (II.16)

- 2: between rotor and stator
  $\begin{align*}
    (13,15) \Rightarrow w_{\theta_2} &= c_{z_2} \tan \beta_2 \\
    (14) \Rightarrow c_{\theta_2} &= U + w_{\theta_2}
  \end{align*}$ (II.17)

- 3: stator outlet
  $\begin{align*}
    (15) \Rightarrow c_{\theta_3} &= c_{z_3} \tan \alpha_3
  \end{align*}$

\[w_{z_3} = c_{z_3}\] (13)
\[c_{\theta_3} = c_i \sin \alpha_i = c_{z_3} \tan \alpha_i\] (15)
Velocity Triangles: Comments

• In axial compressor,
  – the rotor blades increase absolute swirl (push flow downward)
  – the stator blades reduce the swirl (return the flow back towards the axial direction)

Repeated (or Normal) Stage
- velocities at inlet and exit of stage are the same
  \( c_{z3} = c_{z1}, \ c_{\theta3} = c_{\theta1}, \ \alpha_3 = \alpha_1 \)

Zero exit swirl
- no azimuthal velocity at exit of stage \( \alpha_3 = 0 \)

Swirl Velocity Change Example

• Given:
  – Repeating axial flow compressor stage with pitchline radius of 0.50 m, rotational speed of 4050 rpm, inflow axial velocity of 155 m/s and inlet flow angle of 10.2° (in same direction as rotor wheel motion). The rotor blade exit angle is -22.5° and the deviation is 2°.

• Find:
  1. Change in swirl velocity across rotor \( (\Delta c_{01,2}) \)
  2. Change in swirl vel. \( (\Delta c_{0,2,3}) \) across stator

• Assume:
  – axial velocity constant through stage
Swirl Velocity Change Example

- $\Delta c_{\theta,1,2} = ?$
  - from (14): $c_{\theta_1} = c_{z_1} \tan \alpha_1$
  - from (15): $c_{\theta_2} = U + c_{z_2} \tan \beta_2$
  - $c_{\theta_2} - c_{\theta_1} = U + c_{z_2} \tan \beta_2 - c_{z_1} \tan \alpha_1$
  - $\Delta c_{\theta,1,2} = U + c_z (\tan \beta_2 - \tan \alpha_1)$ (II.18)

- $r = 0.50 \text{m}$, $N = 4050 \text{rpm}$
- $c_{z_1} = c_{z_2} = 155 \text{ m/s}$
- $\alpha_1 = 10.2^\circ$, $\alpha_2 = \alpha_1$
- $\chi_3 = 22.5^\circ$, $\delta_m = 2^a$

Swirl Velocity Change Example

- $\Delta c_{\theta,2,3} = ?$
  - $\Delta c_{\theta,2,3} = c_{\theta_3} - c_{\theta_2}$

- $r = 0.50 \text{m}$, $N = 4050 \text{rpm}$
- $c_{z_2} = c_{z_3} = 155 \text{ m/s}$
- $\alpha_2 = 10.2^\circ$, $\alpha_3 = \alpha_1$
- $\chi_3 = 22.5^\circ$, $\delta_m = 2^a$