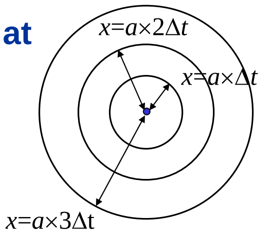
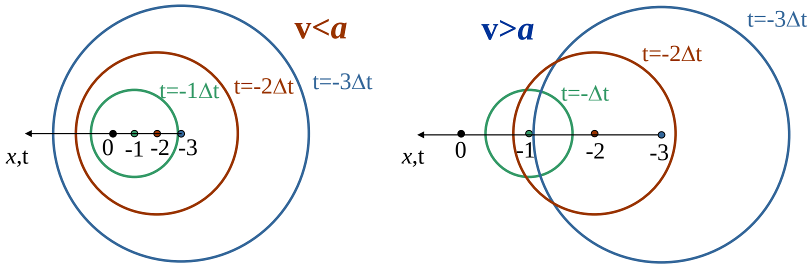
Mach Angle and Mach Number

- Looking for relationship between speed of sound and flow speed (or speed of body moving through fluid)
- Consider small body (point) moving in stagnant fluid
  - continuously produces weak pressure disturbances (e.g., fluid having to go around it)
- Disturbances travel outward spherically at sound speed (a)
- **Look at disturbances generated at equally spaced time intervals**
- Start with body moving with **v** << ***a***
  - e.g., nearly stationary (or moving through incompressible liquid)



Subsonic and Supersonic Motion

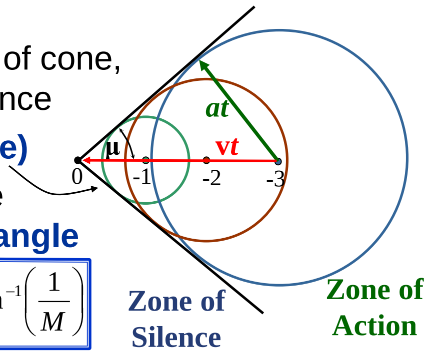
- Now compare two bodies, one moving with **v** < ***a***, subsonic other moving with **v** > ***a***, supersonic



- Subsonic body always behind sound waves launched from previous positions
- Supersonic body moves ahead of previous sound waves

Mach Wave and Mach Angle

- For supersonic flow, can define region where disturbance has had an effect (been "felt/heard")
- Conical region delineated by tangents to sound wave spheres
- Waves coalesce at edge of cone, produce largest disturbance
  - **Mach wave (Mach line)**
- Angle between Mach line and body motion, **Mach angle**



$$\mu = \sin^{-1}\left(\frac{a\cancel{t}}{v\cancel{t}}\right) \qquad \text{(V.A3)} \quad \boxed{\mu = \sin^{-1}\left(\frac{1}{M}\right)}$$

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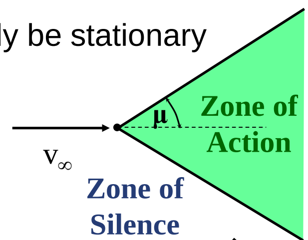
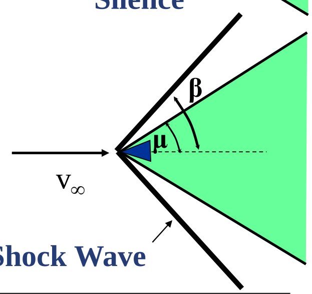
Mach Cone and Shock Waves

- Same behavior holds if we let body be stationary and flow is moving
- Weak disturbances from presence of body
  - can only be felt inside Mach cone
  - can not be felt "upstream"
- What if finite size body? Strong (**nonisentropic**) pressure disturbances can occur, they coalesce to form **shock waves**





will see later that shock angle $\beta > \mu$

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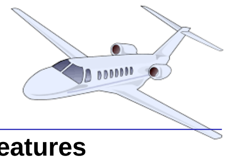
Flow Regimes

- Mach number is often used to provide criterion for defining different flow regimes
- **Subsonic**: $M < 1$; **sonic** $M = 1$; **supersonic** $M > 1$
- A common demarcation for (aerodynamic) flows

$$M_\infty = \mathrm{v}_\infty / a_\infty$$

v_∞

a_∞



Mach Range	Flow Regime	Features
M_∞<0.3	"incompressible"	Δρ < 5% effect
0.3<M_∞<0.8	subsonic	moderate ρ changes with v
0.8<M_∞<1.2	transonic	flow accel can make local M > 1
1.2<M_∞<3	supersonic	stronger ρ changes with v
3<M_∞	hypersonic	very strong shocks

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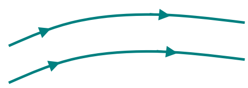
Adiabatic Flow Ellipse

- Another way to look at M effects
- **Energy equation**

$$h_o = h + \frac{\mathrm{v}^2}{2} = const$$



adiabatic/no work stream tube

- **Stagnation T_o** also constant

tpg/cpg
from IV.24

$$T_o = T + \frac{\gamma - 1}{2}\frac{\mathrm{v}^2}{\gamma R} = \mathrm{const}$$

$$\frac{2}{\gamma - 1}\gamma R T + \mathrm{v}^2 = \mathrm{const}$$

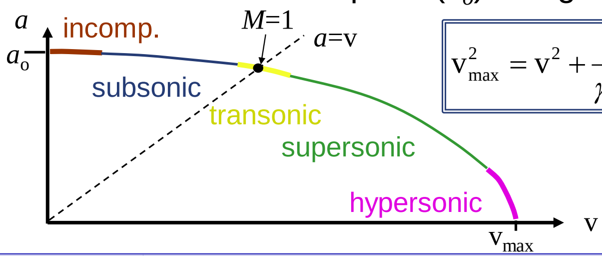
Stagnation speed of sound (no kinetic energy left, v=0)

(V.A4) $$\boxed{\frac{2}{\gamma - 1} a^2 + \mathrm{v}^2 = \mathrm{v}_{\max}^2 = \frac{2}{\gamma - 1} a_o^2}$$

Maximum velocity possible (no thermal energy left, T=0)

Adiabatic Flow Ellipse (con't)

- Transition from low speed (a_o) to high speed (v_{max})



$$\boxed{\mathrm{v}_{max}^2 = \mathrm{v}^2 + \frac{2}{\gamma-1}a^2 = \frac{2}{\gamma-1}a_o^2}$$

Regime	Description/Interpretation
incomp.	$\mathrm{v} \ll a$, $da \ll d\mathrm{v}$, little change in $a(T)$
subsonic	$\mathrm{v} \lesssim a$, M changes primarily to changes in v
transonic	$\lvert \mathrm{v}-a \rvert \ll \mathrm{v},a$
supersonic	$\mathrm{v}>a$, M changes through substantial changes in v and $a(T)$
hypersonic	$\mathrm{v} \gg a$, $d\mathrm{v} \ll da$, M change mostly due to $a(T)$ changes