Mach Angle and Mach Number

- Looking for relationship between speed of sound and flow speed (or speed of body moving through fluid)
- Consider small body (point) moving in stagnant fluid
  - continuously produces weak pressure disturbances (e.g., fluid having to go around it)
- Disturbances travel outward spherically at sound speed ($a$)
- Look at disturbances generated at equally spaced time intervals
- Start with body moving with $v << a$
  - e.g., nearly stationary (or moving through incompressible liquid)

\[ x = a \times 2\Delta t \]
\[ x = a \times \Delta t \]
\[ x = a \times 3\Delta t \]

Subsonic and Supersonic Motion

- Now compare two bodies, one moving with $v < a$, subsonic other moving with $v > a$, supersonic

\[ t = 1\Delta t \quad t = 2\Delta t \quad t = 3\Delta t \]

\[ t = -\Delta t \quad t = -2\Delta t \quad t = -3\Delta t \]

- Subsonic body always behind sound waves launched from previous positions
- Supersonic body moves ahead of previous sound waves
Mach Wave and Mach Angle

- For supersonic flow, can define region where disturbance has had an effect (been “felt/heard”)
- Conical region delineated by tangents to sound wave spheres
- Waves coalesce at edge of cone, produce largest disturbance
  - Mach wave (Mach line)
- Angle between Mach line and body motion, Mach angle
  \[ \mu = \sin^{-1}\left(\frac{a_t}{v_t}\right) \]
  (V.A3) \[ \mu = \sin^{-1}\left(\frac{1}{M}\right) \]

Mach Cone and Shock Waves

- Same behavior holds if we let body be stationary and flow is moving
- Weak disturbances from presence of body
  - can only be felt inside Mach cone
  - can not be felt “upstream”
- What if finite size body?
  Strong (nonisentropic) pressure disturbances can occur, they coalesce to form shock waves
  \[ \beta > \mu \]
  \[ \text{will see later that shock angle} \]

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Flow Regimes

- Mach number is often used to provide criterion for defining different flow regimes
- **Subsonic**: $M < 1$; **sonic** $M = 1$; **supersonic** $M > 1$
- A common demarcation for (aerodynamic) flows

$$M_{\infty} = \frac{v_{\infty}}{a_{\infty}}$$

<table>
<thead>
<tr>
<th>Mach Range</th>
<th>Flow Regime</th>
<th>Features</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{\infty} &lt; 0.3$</td>
<td>&quot;incompressible&quot;</td>
<td>$\Delta p &lt; 5%$ effect</td>
</tr>
<tr>
<td>$0.3 &lt; M_{\infty} &lt; 0.8$</td>
<td>subsonic</td>
<td>moderate $\rho$ changes with $v$</td>
</tr>
<tr>
<td>$0.8 &lt; M_{\infty} &lt; 1.2$</td>
<td>transonic</td>
<td>flow accel can make local $M &gt; 1$</td>
</tr>
<tr>
<td>$1.2 &lt; M_{\infty} &lt; 3$</td>
<td>supersonic</td>
<td>stronger $\rho$ changes with $v$</td>
</tr>
<tr>
<td>$3 &lt; M_{\infty}$</td>
<td>hypersonic</td>
<td>very strong shocks</td>
</tr>
</tbody>
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Adiabatic Flow Ellipse

- Another way to look at $M$ effects
- **Energy equation**

$$h_o = h + \frac{v^2}{2} = \text{const}$$

adiabatic/no work stream tube

- **Stagnation** $T_o$ also constant

$$T_o = T + \frac{\gamma - 1}{2} \frac{v^2}{\gamma R} = \text{const}$$

Stagnation speed of sound
(no kinetic energy left, $v=0$)

$$\frac{2}{\gamma - 1} \frac{\gamma R T + v^2}{\gamma R} = \text{const}$$

$$(V.A4) \quad \frac{2}{\gamma - 1} a^2 + v^2 = v_{\max}^2 = \frac{2}{\gamma - 1} a_o^2$$

Maximum velocity possible
(no thermal energy left, $T=0$)
Adiabatic Flow Ellipse (con’t)

- Transition from low speed ($a_o$) to high speed ($v_{\text{max}}$)

$$v_{\text{max}}^2 = v^2 + \frac{2}{\gamma - 1} a^2 = \frac{2}{\gamma - 1} a_o^2$$

<table>
<thead>
<tr>
<th>Regime</th>
<th>Description/Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>incomp.</td>
<td>$v &lt;&lt; a$, $da &lt;&lt; dv$, little change in $a(T)$</td>
</tr>
<tr>
<td>subsonic</td>
<td>$v \leq a$, $M$ changes primarily to changes in $v$</td>
</tr>
<tr>
<td>transonic</td>
<td>$</td>
</tr>
<tr>
<td>supersonic</td>
<td>$v &gt; a$, $M$ changes through substantial changes in $v$ and $a(T)$</td>
</tr>
<tr>
<td>hypersonic</td>
<td>$v &gt;&gt; a$, $dv &lt;&lt; da$, $M$ change mostly due to $a(T)$ changes</td>
</tr>
</tbody>
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